Time dependent behaviour of an elementary bridge model in presence of uncertainties

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ABSTRACT: During the last decades the excessive deflections of many long span bridges received wide attention both in the research and maintenance fields. The faults of the original previsions can be attributed to (a) lacks in the models adopted for the structural assessment, (b) weak reliability of some shrinkage and creep formulations used for the analyses and (c) differences between the phases of the actual erection techniques and those planned during the design tasks. In this paper the role of the uncertainty affecting quantities and parameters governing the whole attitude of these structures is outlined. With reference to simple but significant examples, it will be shown how some types of bridges possess low sensitivity to the uncertainties, while others result greatly affected by them, exhibiting divergent behaviors over time.

1 INTRODUCTION

During the last decades the excessive deflections of many long span bridges received wide attention both in the research and maintenance fields. The faults of the original previsions can be attributed to (a) lacks in the models adopted for the structural assessment, (b) weak reliability of some shrinkage and creep formulations used for the analyses and (c) differences between the phases of the actual erection techniques and those planned during the design tasks. In this paper, the time dependent behaviour of a simple structure, suitable to model the basic mechanics of cable stayed and cantilever bridges, has been studied. Two elementary structures, one made of a concrete cantilever beam, suspended at the tip by a pretensioned stay, and the other, made of a concrete cantilever beam and post-tensioned through a horizontal cable, have been considered. An introductory deterministic approach recalls how a suitable stay pretensioning or a suitable cable post tensioning, may balance the tip deflection due to selfweight only under elastic hypotheses, without creep and shrinkage effects. According to such computations, the expected attitude of a cable stayed bridge or of a cantilever bridge is governed by the antagonist roles of the selfweight and of the pretensioning actions in the cables. The final value of the deflections results from the difference of two large and opposite numbers, representing downward and upward deflections. Even working with highly accurate computations, such purely elastic results can not be considered fully reliable. In the first part of this work, after some recalls of viscoelastic structural analysis, it will be shown how creep and shrinkage may modify or not modify the elastic results and how these different behaviours evolve with time.

Such results, achieved through conventional assessments, are based on deterministic analyses, which assume that the tensioning forces as well as all the other physical and mechanical factors involved in the structural behaviour, are known and certain data. Actually, these data involve many uncertain quantities, such as environmental factors, material characteristics and pretensioning level in the cables.

Thus, in the second part of the paper, the effects of the uncertainties are studied (Malerba et al. 2011). Two approaches have been considered: a Pure Monte Carlo Approach and a Hybrid Monte Carlo/Possibilistic Approach (Baraldi & Zio 2008). For a given set of data, the creep effects have been handled by means of a step by step computation of the stress history, based on a particular matrix partitioning technique. Through the results, we can distinguish between two different kinds of structures: those which exhibit low sensitivity and those which are greatly affected by the uncertainties of some of the most important parameters which govern the time dependent behaviour.
2 RECALLS OF VISCOELASTIC STRUCTURAL ANALYSIS

In a plane concrete element specimen, for a sustained stress applied at time $t_0$, the total strain at time $t$ is

$$
epsilon_c(t) = \frac{\sigma_c(t_0)}{E_c(t_0)} \left[ 1 + \varphi(t,t_0) \right] = \sigma_c(t_0) \cdot J(t,t_0) \quad (1)$$

where $\varphi(t,t_0)$ is the dimensionless creep coefficient, function of the age of the concrete $t_0$ at loading and of its age $t$ when the strain is measured, and $J(t,t_0)$ is the creep function. Several creep functions have been formulated. The most used are those proposed by ACI (ACI Committee 1982), CEB (CEB 1978, CEB 1984, CEB 1999) and Bažant B3 (Bažant et al. 2000).

Assuming that the principle of superposition holds, the strain corresponding to a stress history composed of two basic stress histories equals the sum of the strains due to each of these. In general, if the stress intensity varies with time, the total strain of the concrete due to the applied stress is given by:

$$
epsilon_c(t) = \frac{\sigma_c(t_0)}{E_c(t_0)} \left[ 1 + \varphi(t,t_0) \right] + \int \frac{\sigma_c(\tau)}{E_c(\tau)} \left[ 1 + \varphi(t, \tau) \right] d\sigma_c(\tau) + \epsilon_{sh}(t,t_0) \quad (2)$$

where $\epsilon_{sh}(t,t_0)$ is the free shrinkage strain and the integral must be understood as a Stieltjes integral. This integral represents the instantaneous strain, plus creep strain due to an increment in concrete stress, gradually introduced during the period $t_0$ to $t$.

2.1 Methods for Structural Analysis

For the sake of the structural analyses and referring to Finite Element type solutions, the creep constitutive law has been developed in various forms in order to compile the stiffness or the compliance matrices of the structure. The most frequently used methods refer to:

- Direct approaches, also called algebraic methods, based on simple quadrature rules of the superposition integral, like the Age Adjusted Effective Modulus Method (A.A.E.M.M.) (Bažant 1972.a, Kríštek et al. 1988, Ghali et al., 2002).
- Steps by step solutions in the two versions:
  - those based on the degenerate kernels and Dirichlet series, which require the knowledge of the last step of computation only (Bažant et al. 1973);
  - those based on the degenerate kernels and Dirichlet series, which require the knowledge of the last step of computation only (Bažant et al. 1973).

In the following, the A.A.E.M. Method will be used for introductory considerations, while the analyses will be carried out through step by step direct integration, based on a matrix partitioning technique, whose details are given in Malerba (Malerba 1986, Malerba 2011).

3 MECHANICS OF AN ELEMENTARY CABLE STAYED BRIDGE

As known, the expected attitude of a cable stayed bridge is governed by the antagonist roles of the selfweight and of the pretensioning of the stays. In the following the relative influence of these two contributions is examined and compared.

3.1 The role of the stays pretensioning

Through a formal demonstration the role of the stay pretensioning is firstly recalled.

3.1.1 Case A. Cantilever beam supported at the tip by a non pretensioned stay

With reference to the elementary cable stayed beam shown in Figure 1, we firstly compute the vertical tip displacement when the stay is not pretensioned.

Through the compatibility equation at the tip $B$, at the loading time $t_0$, we obtain:

$$X(t_0) = \frac{q t_0^3}{3EI} = \frac{u_{10}}{a_{11}^0 + a_{11}^1} \quad (3)$$

For a sustained constant load $p$, the tip deflection of a cantilever beam without any supporting stay, increases with time in affinity to the corresponding concrete creep function. If the tip is hung on a stay, the stay contrasts the downward deflection and, at the same time, keeps increments of the suspending force. At time $t$ the creep effects are computed...
through the A.A.E.M. Method and the compatibility equation becomes:
\[
(1 + \chi \varphi) a_{i1} \cdot X(t) + \varphi(1 - \chi) X(0) \cdot a_{i1} + \\
(1 + \varphi) \cdot u_{10} = -a_{i1} \cdot X(t)
\]
from which we obtain:
\[
X(t) = -\frac{\varphi(1 - \chi) X(0) \cdot a_{i1} - (1 + \varphi) \cdot u_{10}}{(1 + \chi \varphi) \cdot a_{i1}^b + a_{i1}^r}
\]
It is easy to verify that the force in stay increases with time: \( X(t) > X(t_0) \quad (t > t_0) \).

3.1.2 Case B. Cantilever beam supported at the tip by a pretensioned stay

We apply a certain level of pretension to the stay. As shown in Figure 2, the pretensioning is modeled as a relative displacement \( \lambda \) between the end anchorage.

\[ \lambda = u_{10} \cdot a_{i1}^r / a_{i1}^b \]

3.2 Numerical comparisons

The previous cases have been studied through a step by step F.E. numerical analysis based on the creep and relaxation matrix partitioning technique presented in (Malerba 1986, Malerba et al 2011).

The cantilever beam has the double box section and the material characteristics shown in Figure 3. The load \( p \) is set equal to 120 kN/m, the length of the cantilever is \( l = 20 \) m.

![Figure 3. Cantilever beam section.](image)

Figure 3. Cantilever beam section. \( A = 46364 \) cm\(^2\), perimeter = 3510 cm, \( J = 52101743.8 \) cm\(^4\), \( f_{\infty} = 24 \) MPa, \( \varepsilon_{\infty} = -0.00025 \).

Figure 4 shows the time evolution of the force in the stay for the following loading conditions:

1. Cantilever beam under the sustained load of the selfweight. Stay not pretensioned.
2. Weightless cantilever beam under the action of the stay pretensioning only.
3. Cantilever beam loaded by selfweight and supported by the pretensioned stay.

Load condition (1) is the same as the case (A). Creep of the concrete beam causes a continuous grow of the force in the stay. In load condition (2) creep causes the relaxation of the force in the stay, which decreases with time. Load condition (3) is the sum of the previous ones: the total force in the stay is constant and equals the reaction of the rigid support: \( T = 3 / 8q_l \cdot 3 / 8 \cdot 120 kN / m \cdot 20 m = 900 kN \).

![Figure 4. Time evolution of the force in the stay.](image)

Figure 5 shows the time dependent displacements of the stayed cantilever in the two cases. In case (A) the vertical displacement of the tip continues to grow over time, while in case (B) the tip B does not move. Figure 8 shows the comparisons between normalized displacement functions \( u(t)/u(t_0) \) and normalized creep functions \( f(t, t_0) \cdot E(t) \).

The structure as a whole is not homogeneous, but in case (B) the pretensioning of the stay plays the role of a rigid support at tip. As a consequence, the normalized displacement and creep functions coincide while, along the span, the vertical displacements increase with time in affinity to the creep function. Such a structural behaviour corresponds to what previewed by the first theorem of the viscoelasticity, with no variations of the static regime (Fig. 6-7).
CASE A. Cantilever supported at the tip by a non pretensioned stay.

CASE B. Cantilever supported at the tip by a pretensioned stay.

Figure 5. Time dependent displacements of the stayed cantilever.

Figure 6. Time dependent shear forces of the stayed cantilever.

Figure 7. Time dependent bending moments of the stayed cantilever.

Figure 8. Comparisons between normalized displacement function and normalized creep function.
4 MECHANICS OF AN ELEMENTARY PRESTRESSED CANTILEVER BRIDGE

We consider the elementary cantilever bridge model shown in Figure 9. The beam is prestressed by single straight cable, having eccentricity $e$ and prestressing force $N$.

The vertical displacement at the tip B, due to the sustained load $p$, is:

$$u_B = \frac{1}{8} \cdot \frac{p l^4}{EI} \quad (10)$$

The vertical displacement due to prestressing force $N$ is:

$$u_B = \frac{1}{2} \left( \frac{N \cdot e \cdot l^2}{EI} \right) \quad (11)$$

By equating Equation 10 and 11, we obtain that particular value of the prestressing force which, in absence of creep effects, maintains fixed the tip:

$$N = \frac{1}{4} \cdot \frac{p l^2}{e} \quad (12)$$

Creep and shrinkage causes delayed deformations and, in particular, an axial shortening of the beam and the corresponding losses of prestress. The problem has been studied through the step by step F.E. numerical analysis already mentioned. Also in this case three loading conditions have been considered:

1. Cantilever beam under the sustained load of the selfweight. Cable not prestressed.
2. Weightless cantilever beam under the action of the cable prestressing only.
3. Cantilever beam loaded by selfweight and prestressing.

Figure 10 shows the vertical displacements along the span. It should be noted that the tip has not zero displacement and, with time, it slightly moves downward.

Finally, it may be of interest put in evidence separately what happens in the cable with- and without pretensioning. Figure 11 shows the time evolution of the vertical displacements at the tip, for the loading conditions (1) and (2) and for their sum. Curve (3) seems constant with time and thus it tendentially confirms the balancing effect between selfweight and prestressing. In truth, as seen before, the tip moves downward, as one can verify through a careful examination of Figure 12, where the normalized displacement and creep functions are compared. In this case the tip displacement is not exactly affine to the creep function. This is due to the axial shortening, which causes a decrease in the prestressing force.

Figure 10. Time dependent displacements of the prestressed cantilever.

Figure 11. Time evolution of the tip displacement with- and without prestressing.

Figure 12. Comparison between normalized displacement function $u(t)/u(t_0)$ and normalized creep function $f(t,t_0)E_c(28)$. The two curves are not coincident.

5 THE CASE OF A STRUCTURE EVOLVING: FROM A CABLE STAYED TO A PRESTRESSED CANTILEVER BEAM

In the previous sections, with reference to two distinct types of cantilever, one of them cable stayed and the other prestressed with an horizontal cable, the conditions for zero vertical displacement at the tip have been discussed. It has been shown that, in the case of a cantilever supported by a vertical stay, the vertical displacement at the tip under certain conditions results stable along time, while the tip of a prestressed cantilever moves, due to the prestressing losses caused by the axial shortening of the beam.

These concepts are now generalized with reference to a cantilever beam, supported at the tip by a single
stay having a varying inclination. We refer to the system shown in Figure 13. The eight stays slopes listed in Table 1 are considered. The length of the stay is kept constant. The initial cable pretensioning/prestressing force, is that corresponding to the tip zero vertical displacement at time \( t = t_0 \).

![Figure 13. Cable stayed (C.) and prestressed structures (P.1). \( l = 20 \text{ m}, p = 120 \text{ kN/m}. \)](image)

Table 1. Cases studied. Pretensioning in the stays and prestressing in the cable. Forces in [kN].

<table>
<thead>
<tr>
<th>Case</th>
<th>C.1</th>
<th>C.2</th>
<th>C.3</th>
<th>C.4</th>
<th>C.5</th>
<th>C.6</th>
<th>C.7</th>
<th>C.8</th>
<th>P.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>90.00</td>
<td>75.00</td>
<td>60.00</td>
<td>45.00</td>
<td>30.00</td>
<td>15.00</td>
<td>7.50</td>
<td>3.75</td>
<td>0.00</td>
</tr>
<tr>
<td>( T )</td>
<td>900.0</td>
<td>931.0</td>
<td>1039.7</td>
<td>1274.1</td>
<td>1802.7</td>
<td>3483.7</td>
<td>6908.3</td>
<td>13788.0</td>
<td>72000.0</td>
</tr>
</tbody>
</table>

We have seen that, by pretensioning the vertical cable with the force corresponding to the reaction of a rigid support at the tip of the beam, the displacement of the node B results zero and that the first theorem of linear viscoelasticity is confirmed. Now the slope of the generic stay gives rise to an axial compressive force and the consequent shortening of the beam influences the force in the inclined stays. For the cases C.1-C.8 and P1, Figure 14 shows the ratios between time dependent and initial displacements at the tip and a comparison between the displacements evolution and the creep function used in the analyses.

![Figure 14. Ratios between time dependent and initial displacements at the tip and comparison between the displacements evolution and the creep function.](image)

As one can see, for the case C.1, the displacements and creep functions coincide. In the other cases, due to the compression increase and to the shortening of the cantilever beam, these expectations are no longer verified.

6 NON DETERMINISTIC APPROACH

As already said, the environmental, mechanical and technological parameters which govern the cantilever displacements are affected by uncertainties, which are now considered through a non deterministic approach. In this work, we consider uncertain the relative humidity RH [%] and the prestressing force in the cable \( T \) [kN]. For the relative humidity a gaussian distribution with a mean equal to 70% and a standard deviation equal to 10% is used (Figure 16.a). The prestressing force in the cable is related both to the precision of the tensioning devices and on the variability of the human behaviours. For these reasons \( T \) can be considered an epistemic uncertainty and the infinite set of cumulative distributions within a lower and upper cumulative distribution can be employed (Dubois 2006). In particular, \( T \) has been assumed characterized by a possibility distribution with the trapezoidal form shown in Figure 15.b. The corresponding set of cumulative distributions is shown in Figure 16.

![Figure 15. Uncertainty variables considered. (a) Relative humidity (gaussian distribution). (b) Force in the cable (possibilistic distribution)](image)

For the case C.1, the displacements and creep functions coincide. In the other cases, due to the compression increase and to the shortening of the cantilever beam, these expectations are no longer verified.

6.1 Pure Monte Carlo simulation

In a direct probabilistic approach, the possibilistic distribution is transformed into a probabilistic one and the successive propagation of the uncertainties is sampled via MC sampling (Baraldi & Zio 2008). Notice that going from possibility to probability some information is arbitrary introduced in the
transformation procedure which selects one cumulative distribution function (cdf) among the infinite number of cdfs corresponding to the possibility distribution (Figure 16).

The pure MC simulation is set by a single loop, whose operations are:
- sampling of the probabilistic variables;
- processing their uncertainty propagation by the model.

The model used in this work is a finite element code implemented by the authors and called “Creep”. The behaviour of the structure is studied varying the measurement time, from $t = 28$ days to $t = 10000$ days. The number of samples used is $N_c = 3000$.

Figure 17 shows the results in term of vertical displacement of the tip for two cable stayed bridges (C.1 ($\alpha=90^\circ$) and C.5 ($\alpha=30^\circ$)) and for the prestressed bridge (P.1 ($\alpha=0.0^\circ$)).

6.2 Hybrid Monte Carlo and Possibilistic Method

In this second approach, the relative humidity is considered as a probabilistic distribution and the force in the cable as a possibility distribution. The hybrid approach consist in an outer loop and an inner loop:
- outer loop: $N_c$ Monte Carlo sampling of the variables described by probability distribution;
- inner loop: fuzzy interval analysis to process uncertainties described by possibility distribution.

In the inner loop we proceeded by $\alpha$-cuts and, for each $\alpha$-cut, the maximum and the minimum of the output must be found. Since the displacement is a monotonic function of the force $T$, the minimum is associate to left value of the $\alpha$-cut and the maximum to the right value, so it is not necessary to investigate all the values of the $\alpha$-cut, but only its extreme values. The results of this approach are $N_c$ possibilistic distribution of the vertical displacement, one of them plotted in Figure 18.

![Figure 18: Example of one of the possibilistic output of the hybrid approach (Case P.1).](image)

By a special joint aggregation method it is possible to find the upper and the lower cumulative distributions. The results are shown in Figure 20, where it is also plotted the cdf of the previous approach, correctly included in the extreme cdf of the hybrid method. Once again, a diverging behaviour of uncertainties can be observed.

If we chose as a limit state the value of -3 cm (dotted line), the following considerations arise:
1. both cable stayed bridges respect the limit state;
2. for the prestressed bridge we have:
   - by pure MC approach we find a probability of failure equal to 0.1;
   - by hybrid approach we find that the probability of failure is included in the extreme values of 0 (lower cdf) and 0.6 (upper cdf).

This is an interval too large and, in order to improve the estimation of the behaviour of the structure, a better knowledge of the uncertainty in the prestressing force is needed.
4. CONCLUSION

In the first part of this paper, the time dependent behavior of a simple viscoelastic cantilever beam, suspended at the tip by a pretensioned and variously inclined stay, has been studied. The comparisons among the results obtained for the different stay attitudes through a traditional deterministic approach, outline how creep and shrinkage may modify the purely elastic results. In the second part of the paper, through a non deterministic approach, analogous comparisons have been carried out by taking into account the uncertainties associated to the relative humidity percentage and to the intensity of the pretensioning force.

From the structural point of view, such results highlight how the cable stayed scheme exhibits low sensitivity to the considered sources of uncertainty, but, also, how this sensitivity increases with the stay inclination. When the stay/cable is horizontal, as in pretensioned cantilever beam, the uncertainty strongly influences the delayed effects and the structure exhibit behaviours diverging with time.

Comparative diagrams allow to appreciate the effectiveness of this methodology in dealing with problems of structural assessment.

REFERENCES


