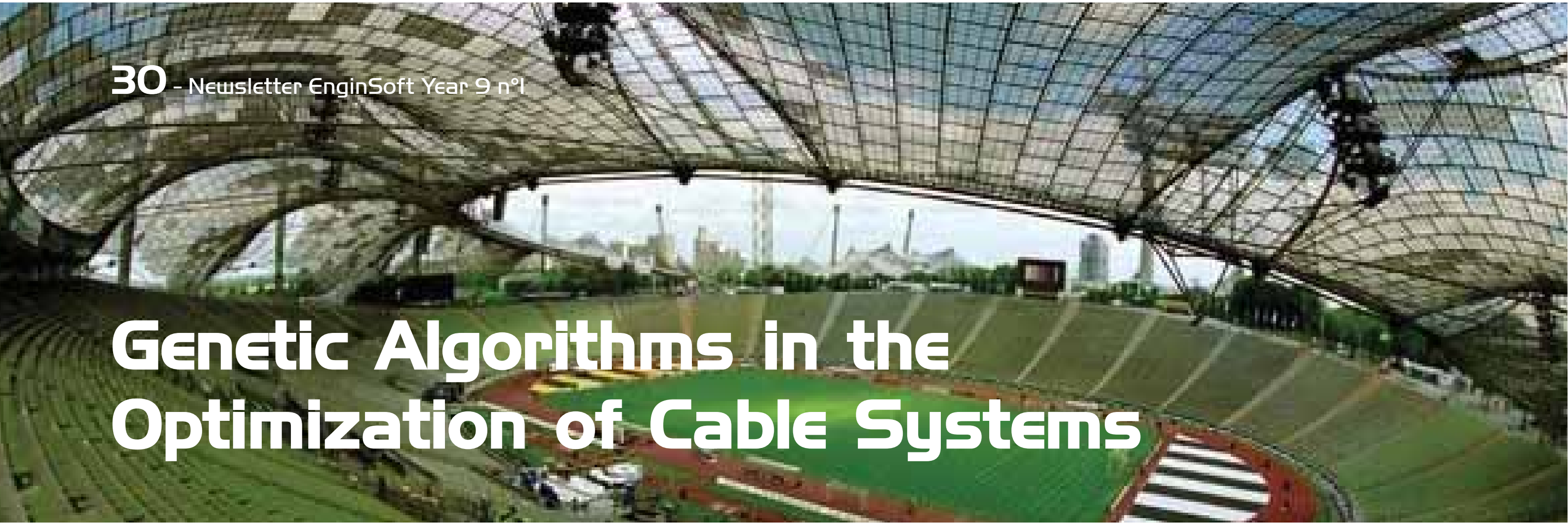


Genetic Algorithms in the Optimization of Cable Systems

In recent years, cable-strut assemblies attract a lot of attention from engineers, due to their versatile shapes, their lightweight and architectural impact. Cable-strut structures have become popular as roofs for arenas, stadiums and sport centers. Yet their working principle is not so easy to understand since they can carry loads thanks to prestress, so that their behavior under loads must be studied taking into account, at least, the geometrical non-linearity. For these reasons, design experience and intuition may not be enough when engineers work with cable systems.

In this article, the matrix theory of a generic three-dimensional pin-jointed structure is first referred. Then, by using modeFRONTIER, a general method able to provide a design solution, which is not only technologically sound, but optimal with respect to the design requirements, is applied to the design of a Geiger Dome.

Introduction

A structure, with a new particular shape cannot be considered as an innovation just because it is very complex. An innovation is a new system working with new mechanical principles, for a better use of materials, lightness etc. Cable-strut assemblies are not an innovation because they are truss systems that are well-known for centuries. However, their last development - the tensegrity systems – can be seen as a real innovation (Motro, 2003). In these systems, the geometrical shape and the prestress in the elements play a crucial role in the structure stability. The first civil structure inspired to the tensegrity principle is the cable dome proposed by Geiger and first employed for the roofs of the Olympic Gymnastics Hall and the Fencing Hall in Seoul (Geiger et al. 1986). The largest existing cable dome is the Georgia Dome designed for the Atlanta Olympics in 1996 (Yuan et al. 2003).

In this article the design of the cable systems is explained, and the Geiger Dome is used as an example.

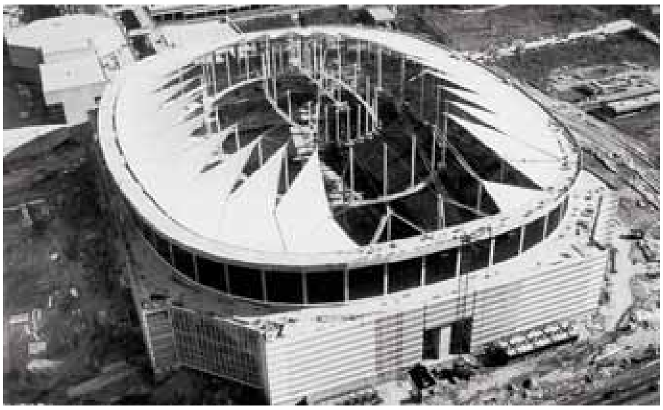


Fig. 1 - The Georgia Dome in Atlanta, U.S.A. reproduced by Tibert, 1999

Particular emphasis is put on a particular optimization procedure, based on a genetic algorithm. This allows us to find a design solution that is not only technologically sound, but optimal in the design requirements (Biondini et al. 2011). The genetic algorithm of the modeFRONTIER software will be used.

Matrix Analysis of Pin-joined frameworks

The matrix formulation is based on Pellegrino and Calladine theories (Pellegrino 1986). The hypotheses are:

a) members are connected by pin-joints;
b) the connectivity between nodes and members is known;
c) self-weight of members is neglected and the additional loads are applied only in the nodes;
d) buckling of the strut is not considered.

Hypotheses a) and c) let the members work only with axial forces, either in compression or tension.

When we consider a generic three-dimensional pin-jointed structure, the equilibrium equations can be written in the following form:

$$At = f \qquad (1)$$

where A is the equilibrium matrix, t the vector of internal forces and f the vector of nodal forces.

In addition, it can be proved through the virtual work principle that the compatibility equation is:

$$A^t d = e \qquad (2)$$

where A^t is the compatibility matrix, d the vector of nodal displacements and e the vector of element elongations.

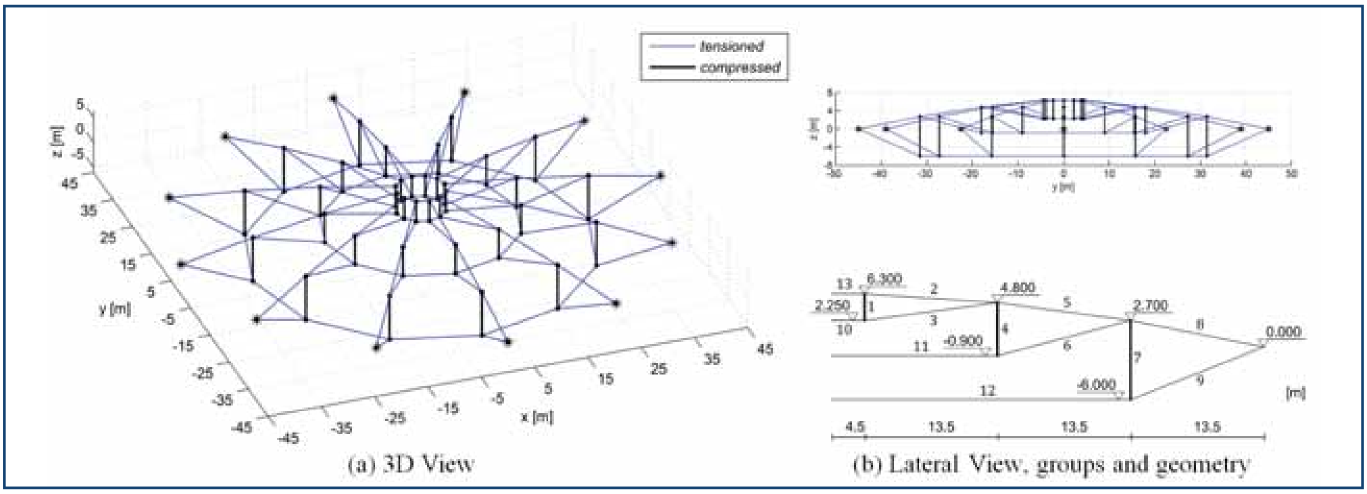


Fig. 2 - Details of the Geiger dome

Through the exploration of the balance subspaces it is possible to classify the pin-jointed framework. In fact, by defining the number of state of self-stress (s) and the number of internal mechanisms (m), we will see the situations listed in Table 1.

	Group no.							
	1	2	3	4	5	6	7	8
Prestress	-0.0433	0.3923	0.2334	-0.0963	0.6268	0.3739	-0.1961	1.0000
	9	10	11	12	13			
Prestress	0.4829	0.4431	0.6979	0.8525	0.7533			

Table 2 - State of self stress of the dome

Type		Static and kinematic properties
I	s = 0 m = 0	Statically determinate and kinematically determinate
II	s = 0 m > 0	Statically determinate and kinematically indeterminate
III	s > 0 m = 0	Statically indeterminate and kinematically determinate
IV	s > 0 m > 0	Statically indeterminate and kinematically indeterminate

Table 1 - Classification of structural assemblies

As pointed out in "Pellegrino 1993", all the information about the assembly can be obtained by the singular value decomposition (SVD) of the equilibrium matrix, whose details are given in "Quarteroni et al. 2008". The state of self-stress is represented by the solutions At = 0, and the mechanism by the solution $A^t d = 0$.

The Geiger Dome

In a Geiger Dome the ridge cables are radially oriented, and the roof is composed of wedge shaped basic units, cyclically distributed around the centre. The Geiger Dome here represented is defined by 84 nodes connected with 156 elements, as shown in fig. 2. The structure is composed of 36 struts and 120 cables. The 12 external nodes are fixed. The symmetry of the dome allows to subdivide the elements into 13 groups, as shown in fig. 2(b).

Given the connectivity and the fixed nodes, the state of self stress can be computed through the singular value decomposition of the equilibrium matrix. The results are reported in table 2. In addition, s=1 and m=61, hence the structure is statically and kinematically indeterminate. However, the self-stress state can stabilize all the internal mechanisms.

Until now, only one balance problem has been solved. In fact, since the vector reported in table 2 is a base, any coefficient Ψ can be chosen, so that Ψ t=0. The precise value of Ψ must consider the performance of the structure under external loads and the resistance of the material. So, for practical purposes, the introduction of additional design criteria, such as structural performance in terms of rigidity and deformability, is needed. This leads us to introduce new variables, such as stress intensity and the actual section of the elements that must also match those of commercial profiles (Biondini et all 2011). The algorithm chosen here is a genetic algorithm implemented in the commercial software modeFRONTIER.

Applied loads and constraints

In addition to the prestress system, two sets of loads are considered:

a) the structural weight;
b) a live vertical load $q=0.5 kN/m_2$, uniformly distributed over the dome.

The constraints of the problem are:

1) a constraint on the maximum dome displacements:

$$d_{max} \leq \frac{l}{250} = \frac{90000}{250} = 360\,mm \qquad (3)$$

2) a constraint on cable resistance: the forces must comply with the resistance FRd provided by the manufacturer reported in Appendix A, with a safety margin $\gamma_{s,i} = 1,5$.
3) a constraint on strut instability:

P_{tot} [ton]	f_{max} [mm] < 360	ψ [N]	P_1	F_2	F_3	P_4	F_5	F_6	P_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}
21.98	-359.57	625638.83	7	23	7	12	16	11	20	18	14	11	16	18	34

Table 3: Optimal solution provided by modeFRONTIER

	Group no.							
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
γ_{safe}	1.40	4.51	1.31	1.42	1.47	1.55	1.51	1.20
	9	*10*	*11*	*12*	*13*			
γ_{safe}	1.62	1.31	1.32	1.41	4.60			

Table 4: Initial safety margins

	Group no.							
	1	2	3	4	5	6	7	8
γ_{safe}	1.41	67.02	1.22	1.10	3.34	1.20	1.10	1.60
	9	10	11	12	13			
γ_{safe}	1.18	1.22	1.02	1.03	68.57			

Table 5 - Final safety margins

$$t_s \le \pi^2 \frac{EI}{l_0^2} \gamma_{s,i}, \quad with \quad \gamma_{s,i} = 1.5 \qquad (4)$$

The first constraint has to be verified under loads, the second and the third have to be verified for both, the prestressing state only and for loads. So, two types of safety margins will be provided: the initial safety margins and the final safety margins.

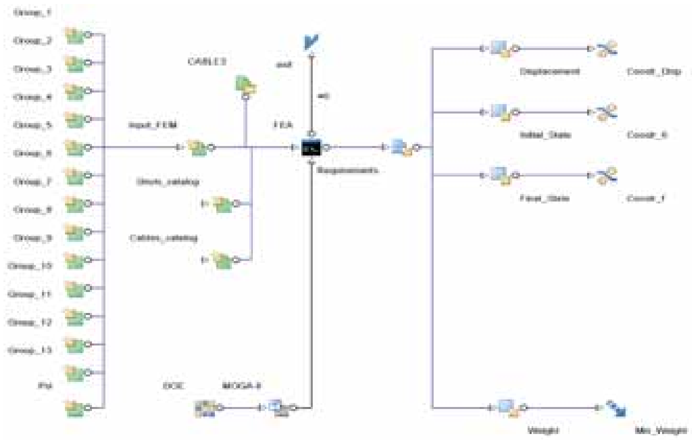


Fig. 3 - Logic and data flow of the optimization process in modeFRONTIER

Representative design variables of the problem

The representative variables of the problem are:

- the coefficient Ψ (1 variable);
- the cable sections divided into groups (10 variables);
- the strut sections divided into groups (3 variables). This means that the optimal solution is searched in a space of 14 variables. The area of the cables has to match those ones of the normalized product. The list of commercial areas considered by the genetic algorithm is reported in Appendix A. Therefore, the algorithm considers 64 different cable types. For the struts, 50 circular sections with the following defined diameter are considered.

$$\Phi = 25 : 5 : 240 \text{ [mm]} \qquad (5)$$

Results of the optimization process

The evaluation of displacements and internal forces in the elements required to assess the fitness of each individual element was made possible by Cable 3, a finite element program implemented in Fortran. The program is able to handle the load response of a general 3D pin-jointed framework considering both mechanical and geometrical non-linearities. The commercial software modeFRONTIER as dealt with the structure optimization problem were p = 40, number of individuals in the population; pc = 0.85, crossover probability; pm = 0.05, mutation probability; elitism disable. The data flow and the logic flow of the modeFRONTIER process are reported in Fig. 3. The optimal solution is shown in Table 3.

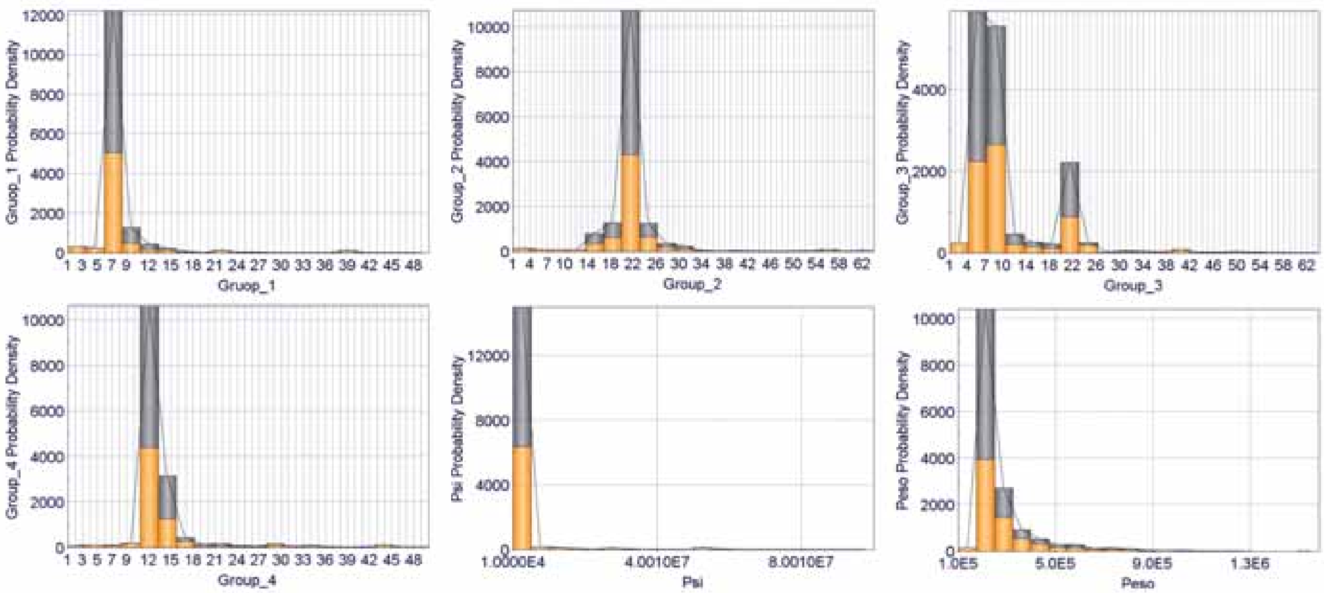


Fig. 4 - Probability density function of some variables

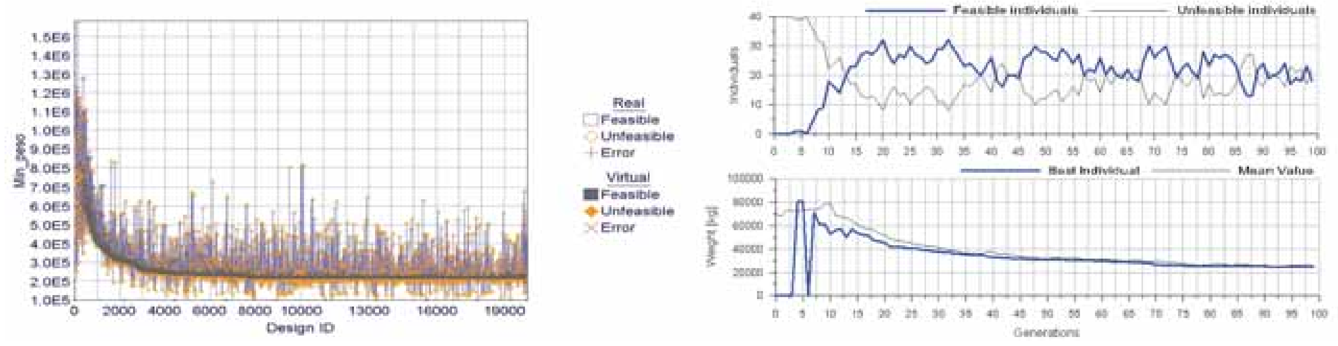
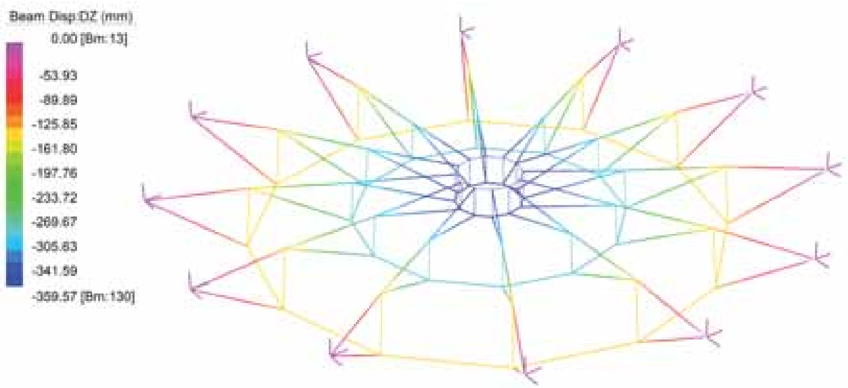
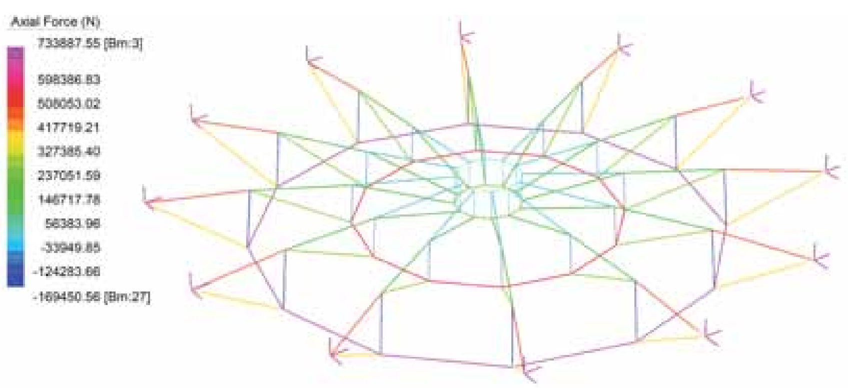


Fig. 5 - Simulation statistics

In Table 4 and 5, the initial and the final safety margin are illustrated. In fact, when dealing with cable systems, not only the final state of the structure (under all loads), but also the initial state (under prestress only) have to be verified. From the tables, we can observe that the dominant condition may be in the initial or in the final state. In fact, there are some elements that increase their force under loads (Fig. 6), while others decrease it, in accordance with the working principle of a cable system.
For the optimal solution, the maximum deflection under loads is equal to -359.57 mm, practically coincident with the allowable maximum deflection set equal to -360 mm.
Figures 10 and 11 show respectively the deformed shape and the axial forces for the optimal solution (Straus7 2004).



(a) Deformed configuration



(b) Final axial forces.

Fig. 6 - Final control of the optimal solution with the commercial FE software Straus7

Conclusion

In this article, an approach to the problem of optimal designs of cable structural systems has been presented. In these systems, the solution of the initial balance problem plays a dominant role. In fact, as these structures work only through axial forces, the geometry and the pretensioning intensity applied to the elements are closely related.
Therefore, the balance configuration must be determined by specific form-finding techniques that provide both the form and the associated stress state. For practical purposes, however, this is not enough and an additional phase, that takes into account the structural performance in terms of rigidity and deformability, is needed. Experiences and intuition may not be sufficient in this second phase because the problem is affected by the geometrical non-linearities and for this reason, a feasible solution may require several trials.

The authors have presented here a general method able to provide a design solution that is not only technologically sound, but optimal with respect to the design requirements.

In the suggested formulation, the solution for the optimization problem has been provided by the genetic algorithm included in modeFRONTIER. This algorithm has been applied to the structural optimization of a Geiger Dome.

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